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# **Multi-static Active Target Tracking using an Invariance Constraint**

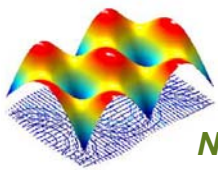
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**Northwest Electromagnetics & Acoustics Research Lab  
Electrical & Comp. Eng. Dept, Portland State Univ.**

Funded by

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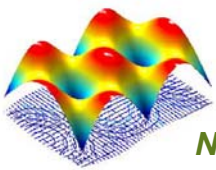


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A significant challenge in tracking targets in multi-static active geometries is the large dimensionality and inherent uncertainty of the track hypothesis space. Traditional tracking approaches (such as Bayesian state estimators) rely on prescribed target kinematics to describe track evolution, but cannot easily incorporate the effects of shallow water multipath. The objective of the proposed research is to improve the capability and robustness of tracking algorithms for Navy multi-static active sonar systems with a physics-based processing technique that relies on the invariance principle and is incorporated into the tracker framework.

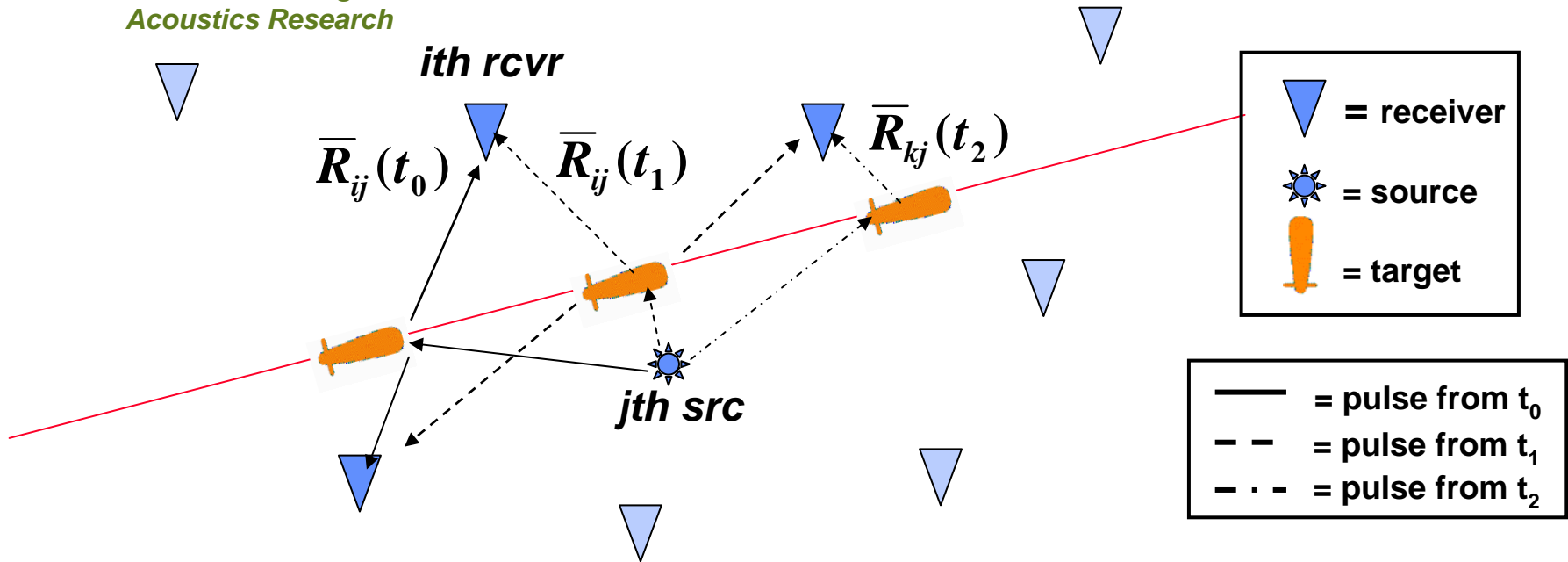
Although the invariance principle is approximately invariant to details of the ocean environment, it still provides a useful relationship between source frequency, frequency offset, target range, and target range rate. For a broadband waveform, the invariance principle suggests a method to constrain the track-hypothesis space by relating the frequency dependent signal characteristics to physically realizable target range rates. This effort is a three year effort (2005-2008).



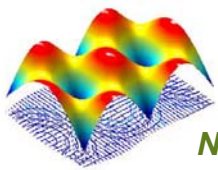
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# Multi-static Sonar Systems



- **Sensor network of underwater acoustic sources and receivers**
  - Acoustic pulses illuminate and scatter from underwater targets
  - Received pulses provide information on time dependent range and Doppler
- **Objective: Determine target track (location vs. time) from observations**
- **Challenges**
  - Underwater propagation physics and bottom reverberation
  - Multi-dimensional solution space

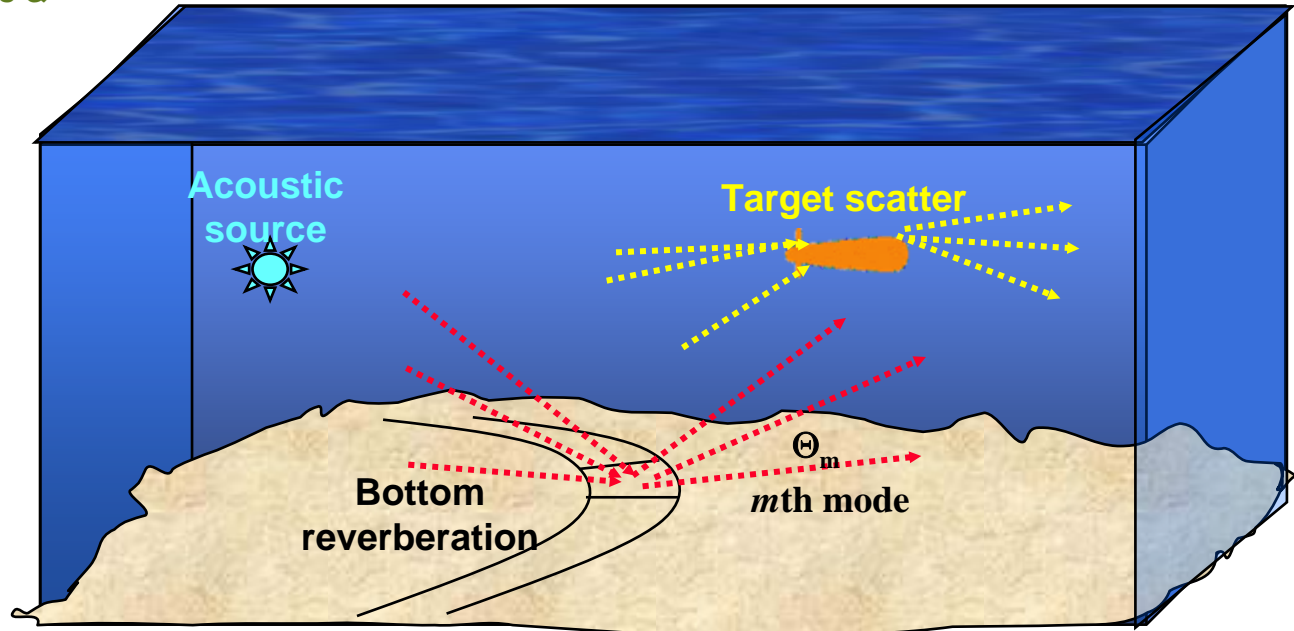
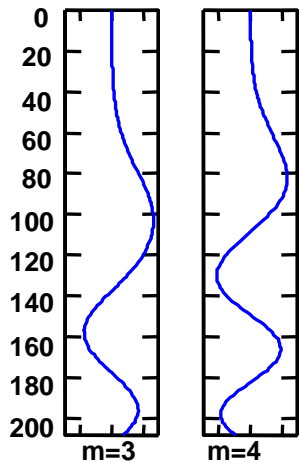
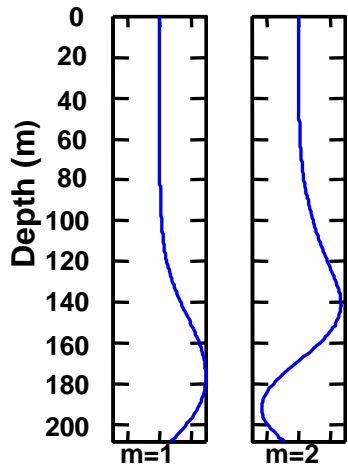


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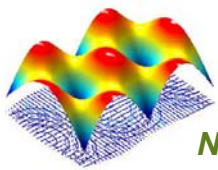
# Active Sonar Signal Contributions

Mode functions  $Z_m(z)$



- **Acoustic waves travel via discrete modes**
  - Environment-dependent propagation paths and velocity (hence multiple arrival times and angles)
- **Reverberation from (rough) ocean bottom**
  - Dominant source of noise for active sonar

*Hypothesis: Moving target time-frequency structure can be separated from reverberation using its invariant structure*

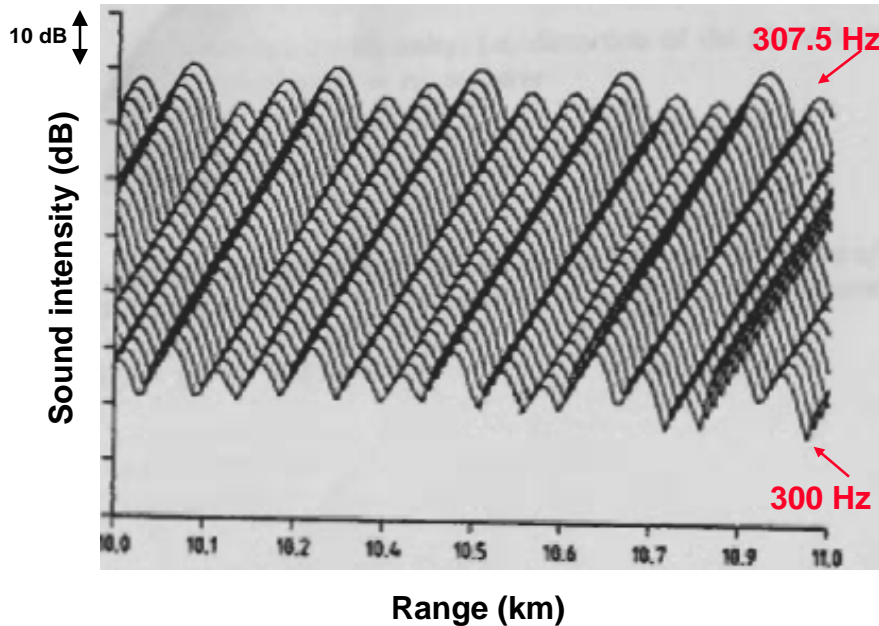


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# Time-Frequency Intensity Variation Invariance Principle

Sound intensity versus range  
(from Brekhovskikh & Lysanov)



Invariant time-frequency structure described by Brekhovskikh in terms of normal mode interference

- Invariance parameter  $\beta$  approximately unity
- Principle applied to interpretation of lofargrams

*Question: is there an invariant structure in active (bi-static) sonar? Can it be exploited?*

SWellEx-3 Single Channel Spectrogram  
JD 204 01:10:00 GMT Elem No.: 1

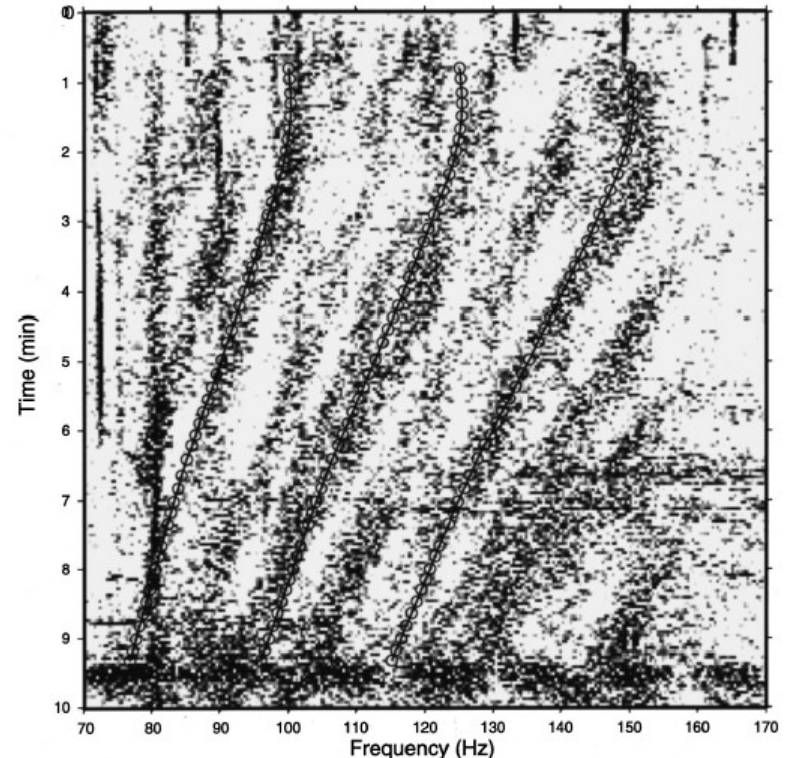
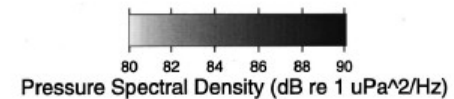
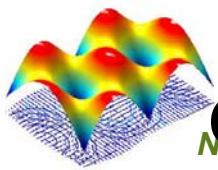


Figure from D'Spain & Kuperman, 1999



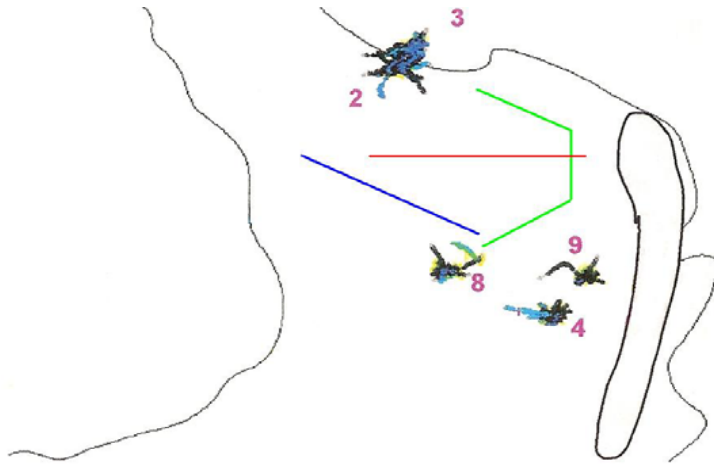


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# Shallow Water Active Classification (SWAC) Characterization & Reduction of Active False Tracks\*

## Geographic Details



### Contacts

- 2=Oil rig
- 3=Moored tanker
- 4=Wreck 1
- 8=Wreck 2
- 9=Wreck 3

### Tracks

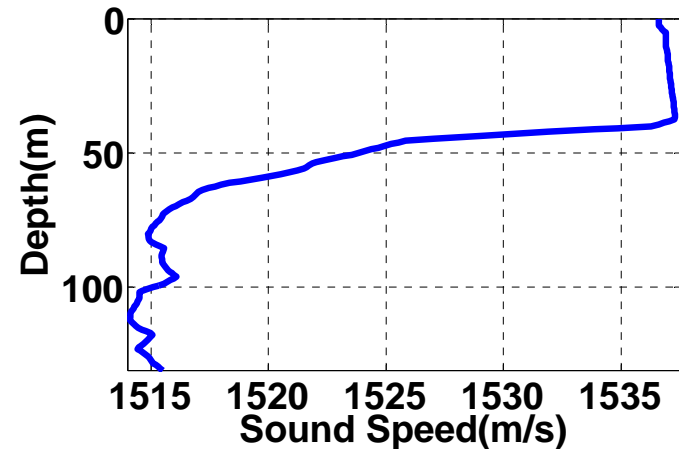
- Red=East/West
- Blue=Diagonal
- Green=Ridge geometry

## Experiment specifications

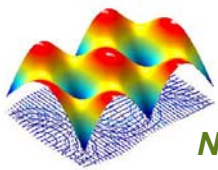
	Variable	Value
Signal processing	Sampling rate	222 to 666 S/s
	Pulse length	1.2 to 2 s
Broadband source	Bandwidth	70 to 400 Hz
	Center frequency	495 to 600 Hz
Kinematics	Receiver depth	66.5 m
	Ship's speed	4.9 knots

## Channel specifications(135 m depth)

Sound speed profile of the channel







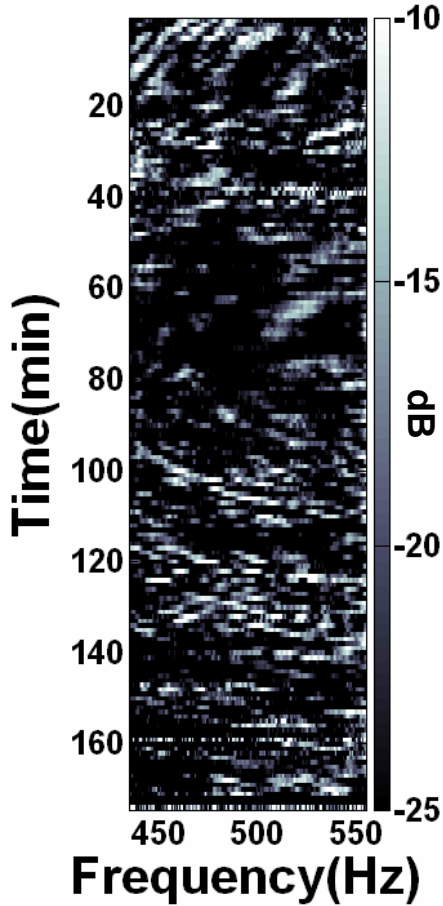
# Active Spectrograms from Malta Plateau

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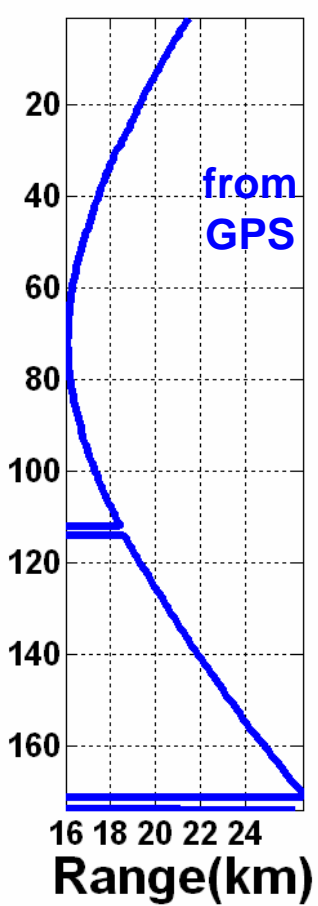
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## Bathtub

### Spectrogram



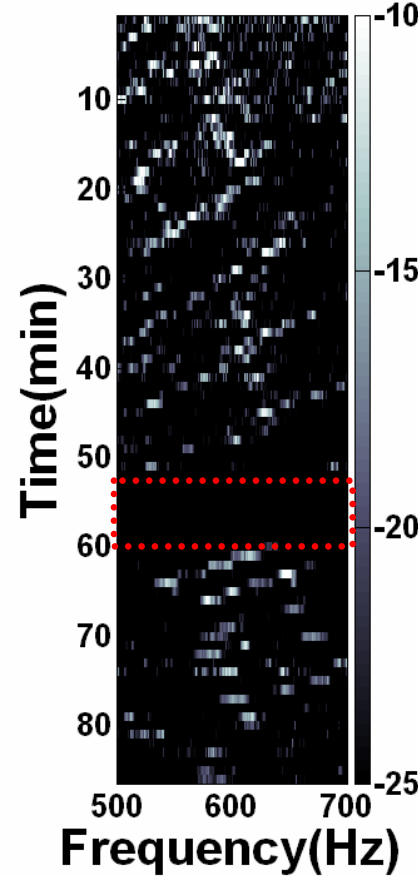
### Range vs time



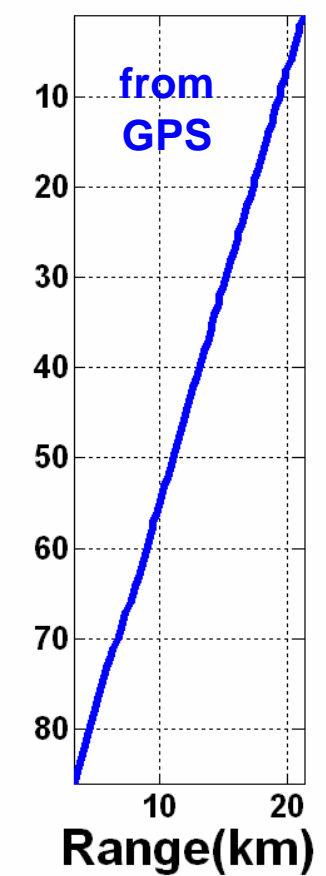
Contact 03/track 56: moored tank

## Linear

### Spectrogram



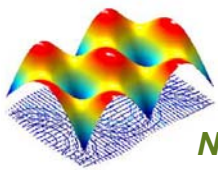
### Range vs time



Contact 09/track 03:Wreck

=missing data

*Note appearance of striation patterns indicative of target track*



# Active Sonar Simulation

- Received (bi-static) pressure:

$$p(r, z, \omega) = C \sum_m \sum_n \left\{ \underbrace{\psi_m(z_s, \omega) \psi_m(z_t, \omega) \frac{e^{ik_m r_1}}{\sqrt{k_m r_1}}}_{\text{Source to target}} \underbrace{G_{mn} \psi_n(z_t, \omega) \psi_n(z_r, \omega) \frac{e^{ik_n r_2}}{\sqrt{k_n r_2}}}_{\text{Target to receiver}} \right\}$$

where:  $\psi_m(z, \omega) = m$ th mode function in the water column.

$k_m =$  horizontal wavenumber of  $m$ th mode

$r_1, r_2 =$  Source/target and target/receiver ranges

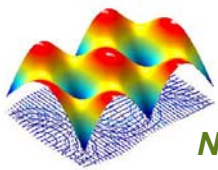
$G_{mn} =$  Scattering matrix defined by the target

$C =$  Normalizing constant

$p(r, z, \omega) =$  Pressure due to a point source of frequency  $\omega$

- Environment values (bathymetry, sound speed, etc.) from NUWC
  - Mode functions computed with KRAKEN
- Scattering matrix: assume no mode coupling (diagonal matrix)

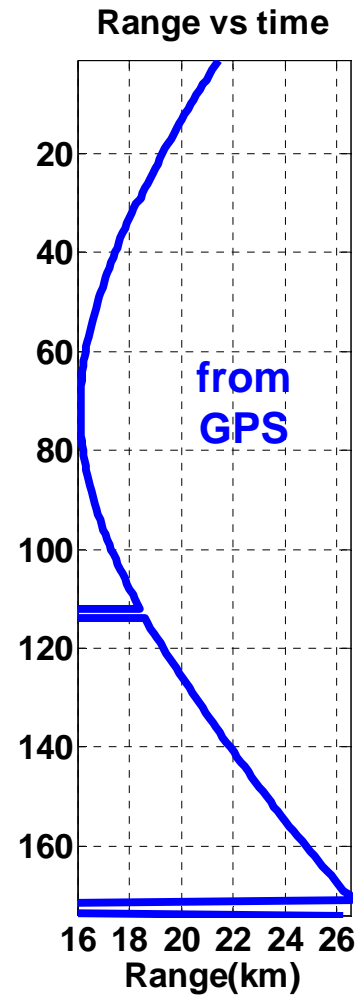
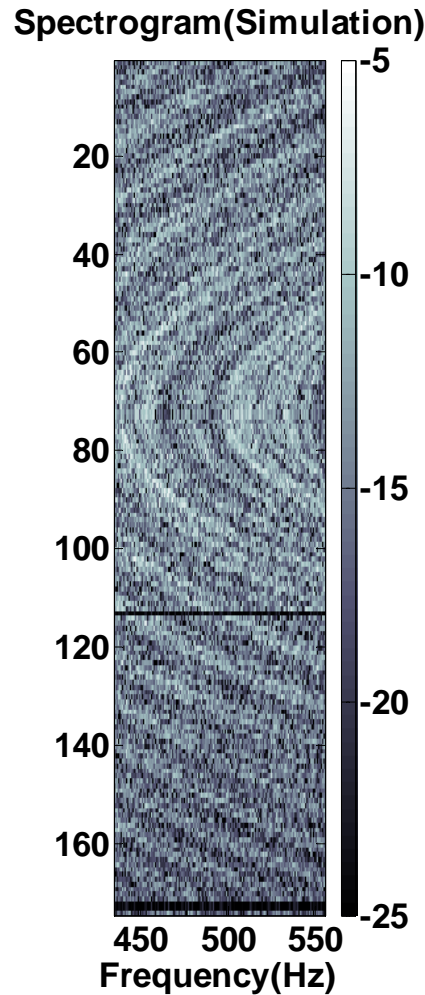
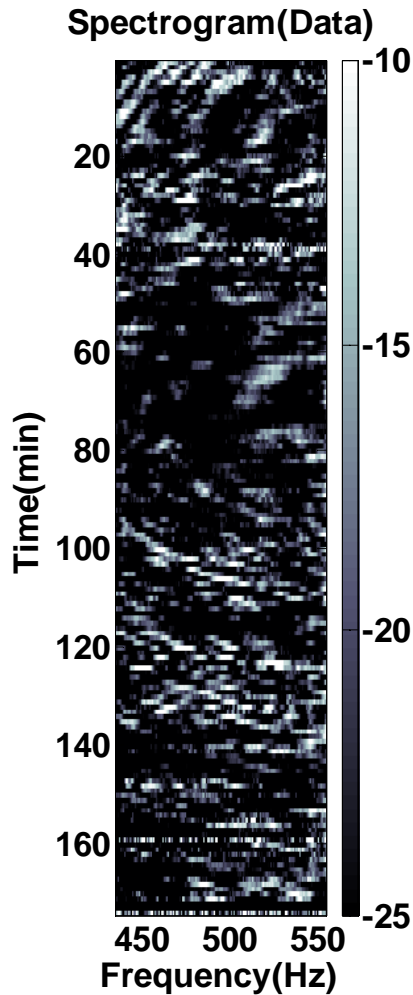




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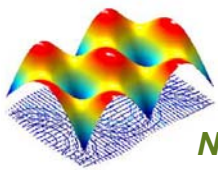
# Measured versus Simulated Spectrograms for Contact 3 (Moored Tank)



Simple scattering matrix: no inter-mode coupling

		m				
n	1	1	0	0	0	...
	0	0	1	0	0	...
	0	0	0	1	0	...
	0	0	0	0	1	...
	⋮	⋮	⋮	⋮	⋮	⋮

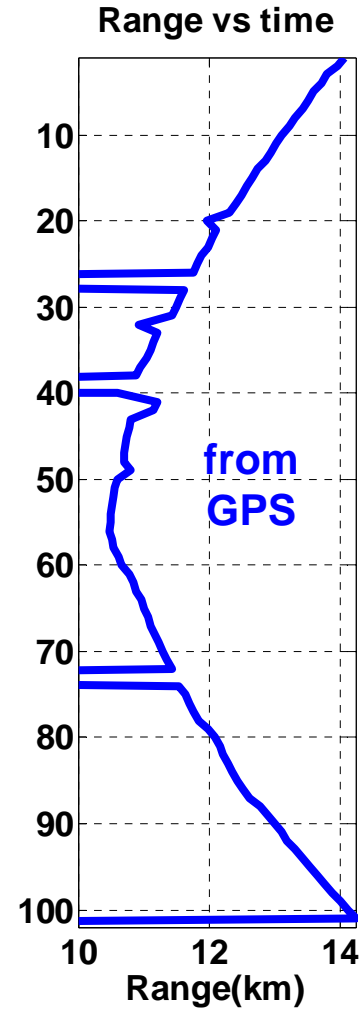
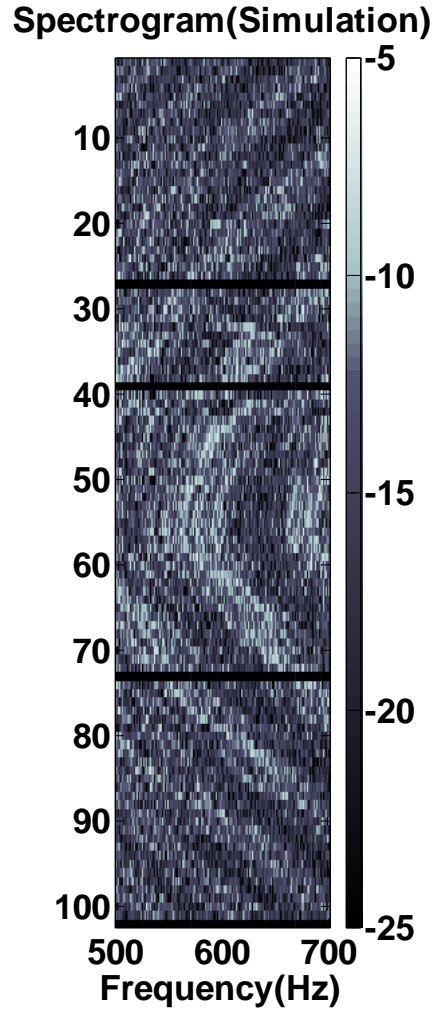
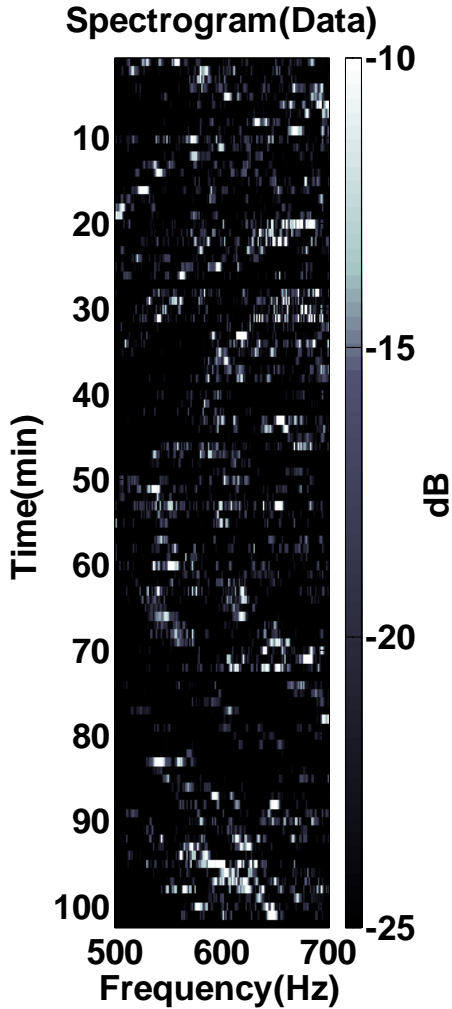
Contact 03/track 56: moored tank



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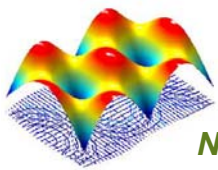
# Measured versus Simulated Spectrograms for Contact 8 (Wreck )



Simple scattering matrix: no inter-mode coupling

	m				
	1	0	0	0	...
	0	1	0	0	...
n	0	0	1	0	...
	0	0	0	1	...
	⋮	⋮	⋮	⋮	⋮

Contact 08/track 23:wreck



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# Tracking using the Invariance Features

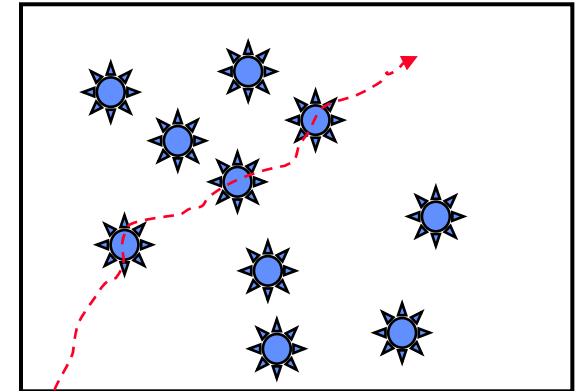
**Standard Kalman Filter (SKF): Tracking based only on kinetics**


State vector

$$X_{state} = \begin{bmatrix} x_t \\ v_t \end{bmatrix}$$

Observations vector

$$V = \begin{bmatrix} range_t \\ bearing_t \end{bmatrix} + \begin{bmatrix} n_{rt} \\ n_{bt} \end{bmatrix}$$



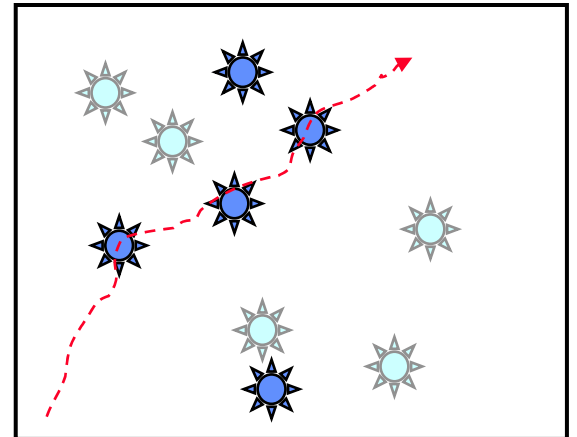
 = detection  
Target track

**Physics-based Invariance with KF: Additional time-frequency constraint imposed to decrease allowable detections**

Def.

Invariant:

$$\frac{\Delta f}{f_{t-1}} = \frac{[\Delta r]}{r_{t-1}} \gamma \quad \longrightarrow \quad f_m(T) = \gamma \frac{f_m(T-1)v_{t-1}T}{r_{t-1}} + 1$$



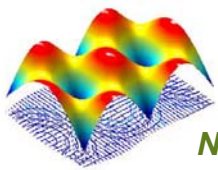
New set of equations:

$$X_{inv} = \begin{bmatrix} x_t \\ v_t \\ f_m(T) \end{bmatrix}$$

$$V_{inv} = \begin{bmatrix} range_t \\ bearing_t \\ f_m(T) \end{bmatrix} + \begin{bmatrix} n_{rt} \\ n_{bt} \\ n_{ft} \end{bmatrix}$$

$$r_t = r_{1t} + r_{2t}$$

$$f_m(t) = \text{Frequency of striation}$$



# Conventional Extended Kalman Filter (CEKF) for Bistatic Geometries

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- State vector (position & velocity)

$$\mathbf{X}_n = [x_n \quad y_n \quad \dot{x}_n \quad \dot{y}_n]^T$$

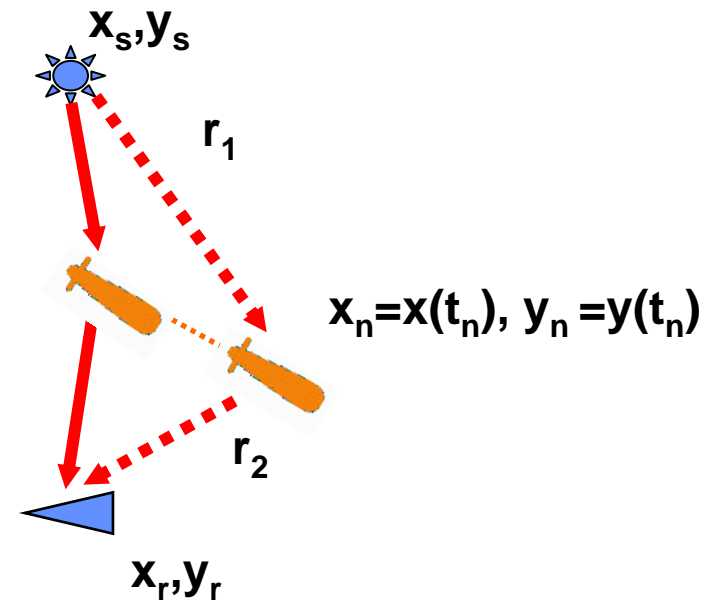
- Nearly constant velocity (NCV) dynamics model

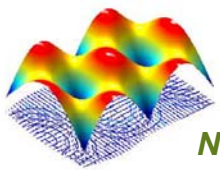
$$\mathbf{X}_{n+1} = \mathbf{F} \mathbf{X}_n + \mathbf{w}_n,$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \Delta t = t_{n+1} - t_n$$

$\mathbf{F}$  = state transition matrix;  $\mathbf{w}_n$  = zero mean, white Gaussian noise

Bistatic geometry:





# CEKF Observations and Measurement Model

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- System measures bistatic range,  $r$ , and bearing angle w.r.t. receiver:

Measurement: 
$$\mathbf{Z}_n = [r_n \quad \phi_n]^T$$

$$\begin{aligned} r_n &= r_1(t_n) + r_2(t_n) \\ &= \sqrt{(x_n - x_s)^2 + (y_n - y_s)^2} + \sqrt{(x_n - x_r)^2 + (y_n - y_r)^2}, \end{aligned}$$

- Measurement model:

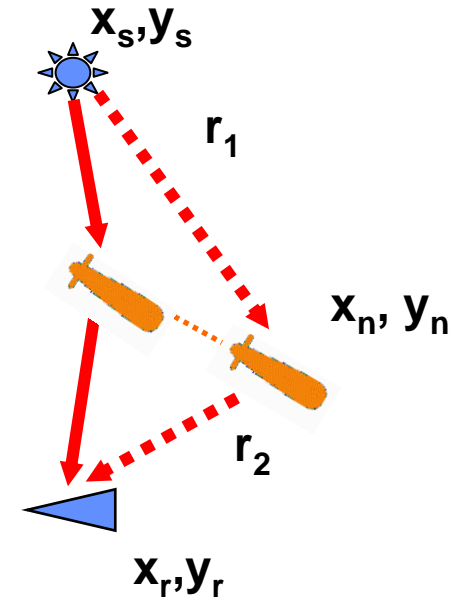
$$\mathbf{Z}_n = h(\mathbf{X}_n) + m_n$$

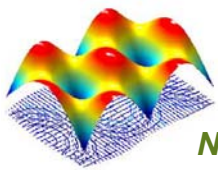
$$h(\mathbf{X}_n) = \begin{bmatrix} r_n \\ \tan^{-1} \frac{y_n - y_r}{x_n - x_r} \end{bmatrix},$$

Measurement noise:

$$m_n = [m_r(t_n), m_\phi(t_n)]^T,$$

Bistatic geometry:





# Invariance EKF (IEKF) State Transition

- State space vector includes time dependent frequency

$$X_n = [x_n \quad y_n \quad \dot{x}_n \quad \dot{y}_n \quad f_n]^T \quad X_{n+1} = F(X_n) + w_n,$$

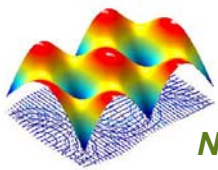
Determine state transition, F, from definition of invariance

$$\frac{\Delta f}{f(T-1)} = \frac{\Delta r}{r(T-1)} \gamma \quad \text{Invariance relation}$$

$$F(X_n) = \begin{bmatrix} x_n + \Delta t \dot{x}_n \\ y_n + \Delta t \dot{y}_n \\ \dot{x}_n \\ \dot{y}_n \\ f_n \left[ 1 + \frac{(\dot{x}_n + \dot{y}_n)\Delta t}{r_n} \right] \end{bmatrix}$$

Relates new frequency  
to previous value  
using invariance





# IEKF Observations & Measurements

- System measures frequency for each pulse
  - For now, assume single value representing the maximum frequency
  - Add to measurement vector

Measurement noise:

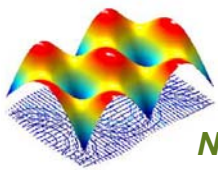
$$\mathbf{Z}_n = [\mathbf{r}_n \quad \phi_n \quad f_n]^T$$

$$\mathbf{m}_n = [m_r(t_n), m_\phi(t_n), m_f(t_n)]^T,$$

- Measurement model becomes:

$$\mathbf{h}(\mathbf{X}_n) = \begin{bmatrix} \mathbf{r}_n \\ \tan^{-1} \frac{\mathbf{y}_n - \mathbf{y}_r}{\mathbf{x}_n - \mathbf{x}_r} \\ f_n \end{bmatrix}$$

- Proceed with Kalman prediction and update as before

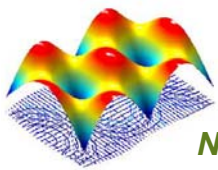


# Tracker Logic

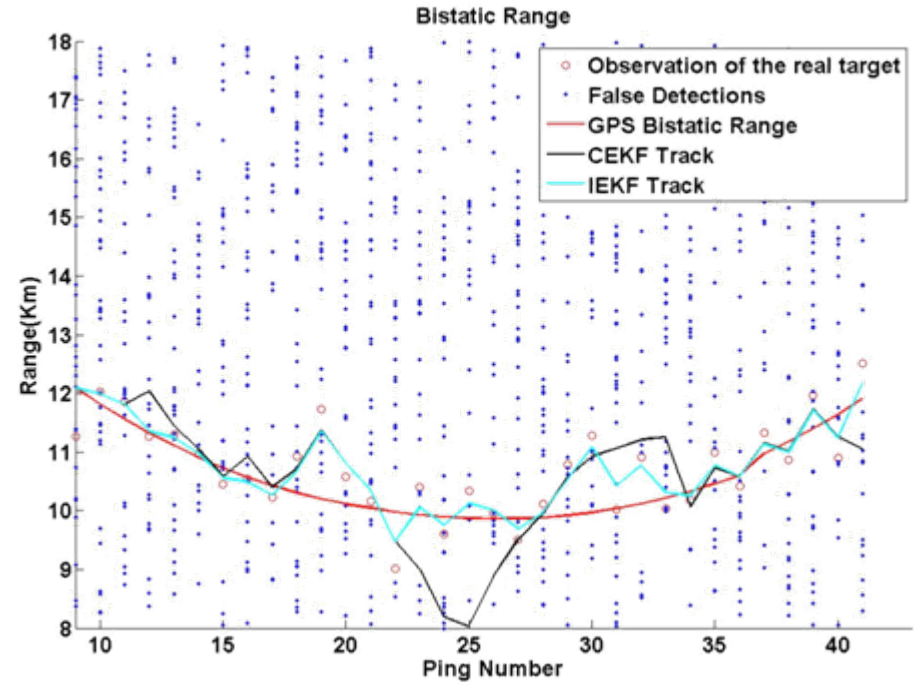
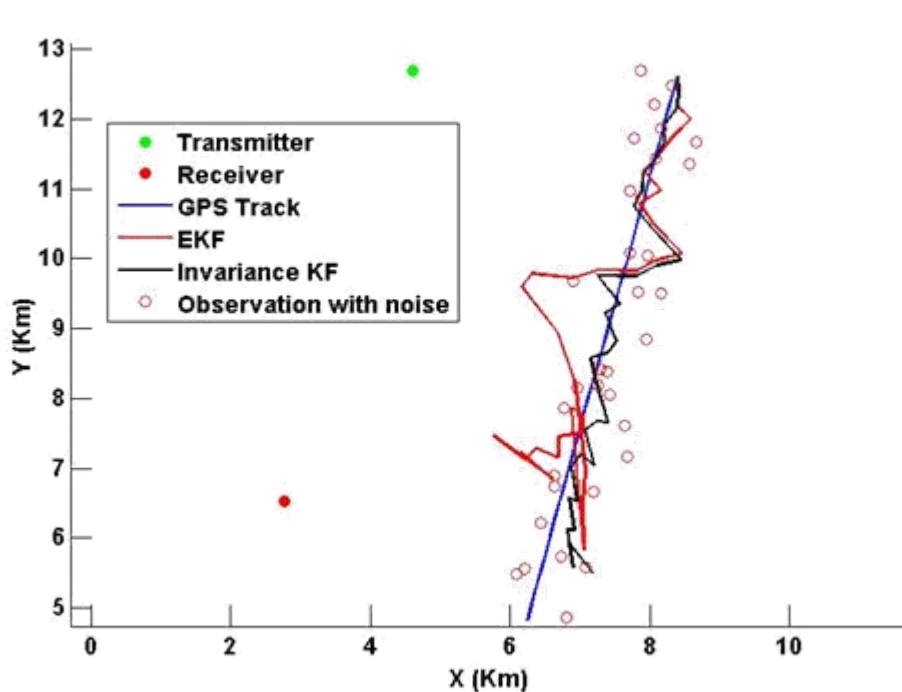
- **Confirm:** If M contacts are associated
- **Discarded:** If less than M contacts are associated with N scans
- **Terminated:** If it's confirmed and after K consecutive missed detections
- **Validated contacts** are those that satisfy the following threshold condition

$$\left( Z_{ij} - c(i)X(i|i-1) \right)' \left( C(i)P(i|i-1)C'(i) + R_i \right)^{-1} \left( Z_{ij} - C(i)X(i|i-1) \right) < \chi^2$$

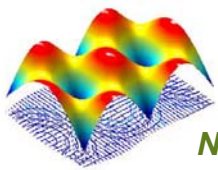
Where the  $\chi^2$  parameter is the association gate parameter



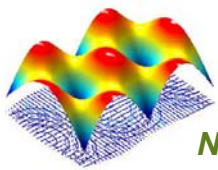
# Tracker Performance



- **Addition of invariance constraint improves tracker performance**
  - Eliminates false detections using frequency (in addition to kinetics)
  - Average range error decreases 34% to 117 m (averaged over 100 realizations)



- **Extend frequency measurement to spectral information**
  - Use family of frequencies representing striations
  - Need transition relationship and estimation (Hough? Radon?)
  - Add uncertainty due to gamma
- **Improve tracker formulation**
  - Multiple tracks, increased tracker logic (track initiation, confirmation, and elimination)
  - More realistic reverberation environment
- **Apply to real data**



# Publications

- J. Quijano, L. M. Zurk, D. Rouseff, Demonstration of the invariance principle for monostatic active sonar, Journal of the Acoustical Society of America, May 2007, submitted for publication
- L.M. Zurk and C. He, *Active target tracking using the bistatic invariance principle*, invited for presentation Acoustical Society of America, Salt Lake City, Utah, June 2007
- L. M. Zurk, J. Quijano, and M. Velankar, D. Rouseff, *Bistatic invariance for active sonar systems*, Acoustical Society of America, Vol. 120, No. 5, p. 3221. November 2006
- J. Quijano, L. M. Zurk, D. Rouseff, *Use of the invariance principle for target tracking in active sonar geometries*, IEEE Oceans Conference, Boston, MA, September 2006
- L.M. Zurk, J. Quijano, D. Rouseff, *Bistatic Invariance Principle for Multi-Static Active Geometries*, Acoustical Society of America, Providence, RI, June 2006
- L. M. Zurk, D. Rouseff, J. Quijano, G. Greenwood, *Bistatic Invariance Principle for Active Sonar Geometries*, European Conference on Underwater Acoustics (ECUA), Carvoviero, Portugal, June 2006